Chapter 7 Numerical Methods

Hints/Solutions

Practice Problems I

1. The given equation is $x^4 + \ell x^3 + mx^2 + nx + 24 = 0$. Let the roots 3, 1, –2 and the fourth root be denoted by β, γ, δ and α respectively.

   The product of the roots = 24

   If the fourth root is α,

   $3(1)(-2)\alpha = 24 \Rightarrow \alpha = -4$

   $\therefore$ The roots of the equation are $\alpha = -4$, $\beta = 3$, $\gamma = 1$ and $\delta = -2$

   $-\ell = (\text{Sum of roots}) = (-4 + 3 + 1 - 2) = -2 \therefore \ell = 2.$

   $m = (\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta)$

   $= \alpha(\beta + \gamma + \delta) + \beta(\gamma + \delta) + \gamma\delta$

   $= (-4)(2) + 3(-1) + 1(-2) = -13$

   $-n = (\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta) = \alpha\beta(\gamma + \delta) + (\alpha + \beta)\gamma\delta$

   $= -12(-1) + (-1)(-2) = 14. \therefore n = -14$

   $\ell + m - n = 2 - 13 + 14 = 3$

   Hence, the correct option is (D).

2. The given equation is $x^3 + 5x^2 - 12x - 36 = 0$ (9)

   Let the roots be α, 3α and β.

   $\therefore \quad 4\alpha + \beta = -5$ (10)

   $3\alpha^2 + 4\alpha\beta = -12$ (11)

   and $3\alpha^2\beta = 36$ (12)

   Combining (10),(12) would produce a cubic equation, while combining (10), (11) would produce a quadratic.

   (10), (11) $\Rightarrow 3\alpha^2 + 4\alpha(-5 - 4\alpha) = -12$

   $\Rightarrow 13\alpha^2 + 20\alpha - 12 = 0 \Rightarrow (\alpha + 2)(13\alpha - 6) = 0.$

   $\therefore \alpha = -2$ or $6/13$

   $\therefore$ Let us find β in each case from (10) When $\alpha = -2$, $\beta = 3$ when $\alpha = 6/13$, $\beta = -89/13$. Only in the first case, (12) is satisfied.

   $\therefore \alpha = 2$ and $\beta = 3$ i.e. the third root is 3.

   $\therefore$ The third root is 3

   Hence, the correct option is (B).

3. $f(x) = x^6 + 5x^5 + 11x^4 + 25x^3 + 34x^2 + 20x + 24 = 0.$

   There are no changes of sign in $f(x)$, $f(x) = 0$ has no positive roots given $f(x) = 0$ has four complex roots $f(x) = 0$ has two negative roots. The number of sign changes in $f(-x)$ has to be more than 2 by an even number. In fact there are four sign changes in $f(-x)$.

   Hence, the correct option is (D).

4. Let $f(x) \equiv x^5 + 5x^4 - 103x^3 - 329x^2 + 2802x + 3024 = 0$

   $f(x)$ has two sign changes

   $\therefore f(x) = 0$ has 2 or 0 positive roots

   But it is given that it has one positive root. With this we conclude that $f(x) = 0$ has two positive roots.

   $f(-x) = -x^5 + 5x^4 + 103x^3 - 329x^2 - 2802x + 3024 = 0$

   $f(-x)$ has 3 sign changes

   $\therefore f(-x) = 0$ has 3 or 1 negative roots.

   But it is given that $f(x) = 0$ has two negative roots. With this we conclude that $f(x) = 0$ has 3 negative roots

   $\therefore$ All the five roots are accounted for.

   $\therefore f(x) = 0$ has zero non-real roots.

   Hence, the correct option is (A).

5. $f(x) = 3x^4 - 13x^3 + 7x^2 + 17x + a - 10 = 0$ has 3 positive roots. The number of sign changes in $f(x)$ have to be 3, 5, ….. as $f(x)$ is a 4th degree polynomial there have to be exactly 3 sign changes a – 10 must be negative.

   Among the options 4 is less than 10.

   Hence, the correct option is (B).

6. Given $f(x) = x^3 - x - 5$

   We know that $f(0) = -5 < 0$ and $f(3) = 27 - 3 - 5 = 19 > 0$

   $\therefore$ One root lies between 0 and 3 and the first approximation is $\frac{0+3}{2} = 1.5$

   Also $f(1.5) < 0$ and $f(3) > 0$

   $\therefore$ The second approximation is $\frac{1.5+3}{2} = 2.25$

   Now, $f(2.25) = (2.25)^3 - 2.25 - 5 > 0$

   $\therefore$ The third approximation x_2 is $\frac{1.5+2.25}{2}$

   $= \frac{3.75}{2} = 1.875$

   Hence, the correct option is (B).

7. Let $f(x) = 2x - \cos x$

   $f(0.5) = (0.5 \times 2) - \cos(0.5) = 1 - 0.8775 = 0.1224 > 0$

   and $f(0) = 0 - \cos 0 = -1 < 0$

   $\therefore$ The root lies between 0 and 0.5

   The first approximation to the required root

   $= \frac{0+0.5}{2}$

   $= 0.25$

   Now $f(0.25) = 2(0.25) - \cos(0.25)$

   $= 0.5 - 0.9689$

   $= -0.4689 < 0$

   $\therefore$ The root lies between 0.25 and 0.5

   $\therefore$ The second approximation to the required root

   $= \frac{0.25 + 0.5}{2} = \frac{0.75}{2} = 0.375$

   Hence, the correct option is (C).

8. Let $f(x) = x^3 + x - 11$

   $f(2) = 8 + 2 - 11 = -1 < 0$ and

$f(3) = 27 + 3 - 11 > 0$

$\therefore$ One root lies between 2 and 3

The first approximation to the root, by bisection method

$$= \frac{2+3}{2} = 2.5$$

Now, $f(2.5) = (2.5)^3 + 2.5 - 11 = 7.125 > 0$

$\therefore$ The root lies between 2 and 2.5

$\therefore$ The second approximation to the root

$$= \frac{2+2.5}{2} = \frac{4.5}{2} = 2.25$$

Now $f(2.25) = (2.25)^3 + 2.25 - 11 = 2.6406 > 0$

$\therefore$ The root lies between 2 and 2.25

$\therefore$ The third approximation to the root

$$= \frac{2+2.25}{2} = \frac{4.25}{2} = 2.125$$

Now $f(2.125) = (2.125)^3 + 2.125 - 11 = 0.7207 > 0$

$\therefore$ The root lies between 2 and 2.125

$\therefore$ The fourth approximation to the root is

$$\frac{2+2.125}{2} = \frac{4.125}{2} = 2.0625$$

Hence, the correct option is (D).

9. Standard result

Hence, the correct option is (B).

10. Let $f(x) = x^3 - x - 4$

$f(1) = 1 - 1 - 4 = -4 < 0$ and

$f(2) = 8 - 2 - 4 > 0$

$\therefore$ One root lies between 1 and 2.

Given 1.666, 1.780 are first two approximates $f(1.780) < 0$ and $f(2) > 0$

$\therefore$ The root lies between 1.780 and 2

$\therefore$ The third approximation is

$$x_2 = \frac{x_1\ f(b) - b\ f(x_1)}{f(b) - f(x_1)}$$

as $f(2) = 2$ and $f(1.780) = -0.1402$

$$x_2 = \frac{(1.780)(2) - 2\times(-0.1402)}{2-(-0.1402)} = 1.794$$

Hence, the correct option is (B).

11. Let $f(x) = 2x - 3\sin x - 5$

$f(2) = 4 - 3\sin 2 - 5 = -3.7278 < 0$

$f(3) = 6 - 3\sin 3 - 5 = 0.5766 > 0$

$\therefore$ A root lies between 2 and 3

Here $a = 2$, $b = 3$

$$\therefore x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{2(0.5766) - 3(-3.7278)}{0.5766 - (-3.7278)}$$

$$= \frac{1.1532 + 11.1834}{4.3044} = 2.866$$

Now $f(2.866) = 2(2.866) - 3(\sin 2.866) - 5$
$= -0.0843 < 0$

and $f(3) > 0$

$\therefore$ The root lies between 2.866 and 3

$$\therefore x_2 = \frac{2.866(0.576) - 3(-0.0843)}{0.5766 - (-0.0843)}$$

$$= \frac{1.6508 + 0.2529}{0.6609} = 2.8804$$

Hence, the correct option is (C).

12. Let $f(x) = x^2 - 2\log_e x - 10$

$f(3) = 9 - 2\log_e 3 - 10 = -3.19722 < 0$

$f(4) = 16 - 2\log_e 4 - 10 = 3.2274 > 0$

$\therefore$ A root lies between 3 and 4

Here $a = 3$, $b = 4$

$\therefore$ The first approximation $x_1 = \dfrac{af(b) - bf(a)}{f(b) - f(a)}$

$$= \frac{3\times(3.2274) - 4\times(-3.1972)}{(3.2274) - (-3.1972)}$$

$$= \frac{9.6822 + 12.7888}{6.4246}$$

$$= \frac{22.471}{6.4246} = 3.4976$$

Now $f(x_1) = f(3.4976) = -0.2709 < 0$ and $f(4)$ is positive

$\therefore$ The root lies between 3.4976 and 4

The second approximation to the root is given by

$$x_2 = \frac{3.4976(3.2274) - 4(-0.2709)}{3.2274 - (-0.2709)}$$

$$= \frac{11.2881 + 1.0836}{3.4983} = \frac{12.3717}{3.4983} = 3.5364$$

Now $f(3.5364) = -0.0200 < 0$

$\therefore$ The root lies between 3.5364 and 4

The third approximation to the root

$$x_3 = \frac{(3.5364)(3.2274) - 4(-0.02)}{3.2274 - (-0.02)}$$

$$= \frac{11.41337736 + 0.08}{3.2474} = 3.5392$$

Hence, the correct option is (A).

13. Standard result.

Hence, the correct option is (A).

14. We know that Newton iteration form for $\sqrt{b}$ is

$$x_{i1} = \frac{1}{2}\left(x_i + \frac{b}{x_i}\right)$$

Given $x_0 = 5.5$, $b = 28$

$$\therefore x_1 = \frac{1}{2}\left[x_0 + \frac{28}{x_0}\right]$$

$$= \frac{1}{2}\left[5.5 + \frac{28}{5.5}\right]$$

$$x_1 = 5.29545$$

Hence, the correct option is (A).

15. We know that the Newton's iterative formula to find $\sqrt[b]{a}$ is

$$x_{i+1} = \frac{1}{b}\left\{(b-1)x_i + \frac{a}{x_i^{b-1}}\right\}$$

Put $b = 3$ and $a = N$, we get

$$x_{i+1} = \frac{1}{3}\left\{2x_i + \frac{N}{x_i^2}\right\}$$

Hence, the correct option is (C).

16. We know that, the Newton's iterative formula for $\sqrt[b]{a}$ is

$$x_{n+1} = \frac{1}{b}\left\{(b-1)x_n + \frac{a}{x_n^{b-1}}\right\}$$

$\therefore$ To find the $\sqrt[3]{24}$, put $b = 3$ and $a = 24$

Let $x_0 = 2.5$

$$x_1 = \frac{1}{3}\left\{2x_0 + \frac{24}{x_0^2}\right\}$$

$$= \frac{1}{3}\left(2(2.5) + \frac{24}{(2.5)^2}\right)$$

$$= \frac{1}{3}\left\{5 + \frac{24}{6.25}\right\} = \frac{1}{3}\{8.84\} = 2.946$$

$$x_2 = \frac{1}{3}\left\{2x_1 + \frac{24}{x_1^2}\right\}$$

$$\therefore x_2 = \frac{1}{3}\left\{2(2.946) + \frac{24}{(2.946)^2}\right\}$$

$$= \frac{1}{3}\left(5.892 + \frac{24}{2.6789}\right)$$

$$= 2.8857 \cong 2.885$$

Hence, the correct option is (B).

17. Standard result

Hence, the correct option is (B).

18. We have by Newton-Raphson method

$$x_{n+1} = x_n\,(2 - x_n N)$$

Let the initial approximation be 0.045. Then

$$x_1 = x_0\,(2 - 22x_0) = (0.045)\,(2 - (22)\,(0.045))$$
$$= 0.04545$$
$$x_2 = x_1\,(2 - 22x_1) = (0.04545)\,(2 - (22)(0.04545))$$
$$= 0.0454545$$

Since $x_1 = x_2$, the reciprocal of 22 is 0.0454545.

Hence, the correct option is (A).

19. Let $f(x) = x^3 - 3x - 5$

Then $f^1(x) = 3x^2 - 3$

We know that, the Newton's iterative formula is

$$x_{i+1} = x_i - \frac{f(x_i)}{f^1(x_i)}$$

Given $x_0 = 2$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f^1(x_0)}$$

$$= 2 + \frac{1}{3} = 2.333$$

Hence, the correct option is (C).

20. Let $f(x) = x^4 + x^3 - 7x^2 - x + 5$

$f^1(x) = 4x^3 + 3x^2 - 14x - 1$

Let $x_0 = 2$

$$\therefore x_{n+1} = x_n - \frac{f(x_n)}{f^1(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f^1(x_0)}$$

$$= 2 - \frac{(-1)}{4(8)+3(4)-28-1} = 2 + \frac{1}{32+12-29}$$

$$= 2 + \frac{1}{15} = 2.0666$$

Hence, the correct option is (A).

21. Given:

$x + 2y + 3z = 16$

$3x + 5y + 8z = 43$

$4x + 9y + 10z = 57$

Step 1:

The matrix equation is $\underset{A}{\begin{bmatrix} 1 & 2 & 3 \\ 3 & 5 & 8 \\ 4 & 9 & 10 \end{bmatrix}}\underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{B}{\begin{bmatrix} 16 \\ 43 \\ 57 \end{bmatrix}}$

Let $LU = A$

$$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix}\begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 5 & 8 \\ 4 & 9 & 10 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23} \\ l_{31}u_{11} & l_{31}u_{12}+l_{32}u_{22} & l_{31}u_{13}+l_{32}u_{23}+u_{33} \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 5 & 8 \\ 4 & 9 & 10 \end{pmatrix}$$

Equating the corresponding elements of the first row,

$\boxed{u_{11}=1,}$ $\boxed{u_{12}=2,}$ $\boxed{u_{13}=3}$

Equating the corresponding elements of the second row,

$l_{21}u_{11}=3 \Rightarrow \boxed{l_{21}=3}$

$l_{21}u_{12}+u_{22}=5 \Rightarrow 3\,(2)+u_{22}=5 \Rightarrow u_{22}=-1$

$l_{21}u_{13}+u_{23}=8 \Rightarrow (3)\,(3)+u_{23}=8 \Rightarrow u_{23}=-1$

Equating the corresponding elements of the third row,

$l_{31}u_{11}=4 \Rightarrow \boxed{l_{31}=4}$

$l_{31}u_{12}+l_{32}u_{22}=9 \Rightarrow (4)\,(2)+l_{32}\,(-1)=9$

$\Rightarrow -l_{32}=9-8=1$

$\boxed{l_{32}=-1}$

$l_{31}u_{13}+l_{32}u_{23}+u_{33}=10$

$\Rightarrow (4)\,(3)+(-1)\,(-1)+u_{33}=10$

$\Rightarrow u_{33}=10-12-1=-3$

$\boxed{u_{33}=-3}$

Thus $L=\begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & -1 & 1 \end{pmatrix}$ and $U=\begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -1 \\ 0 & 0 & -3 \end{pmatrix}$

Step 2:

$LUX=B \Rightarrow LY=B$, where $UX=Y$

$$LY=B \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & -1 & 1 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 16 \\ 43 \\ 57 \end{pmatrix}$$

$\Rightarrow \boxed{y_1=16,}\ 3y_1+y_2=43 \Rightarrow 48+y_2=43 \Rightarrow \boxed{y_2=-5}$

$4y_1-y_2+y_3=57 \Rightarrow 64+5+y_3=57 \Rightarrow y_3=57-69$

$\Rightarrow y_3=-12$

Step 3:

$UX=Y$

$$\Rightarrow \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -1 \\ 0 & 0 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 16 \\ -5 \\ -12 \end{pmatrix}$$

$\Rightarrow x+2y+3z=16$

$-y-z=-5$

$-3z=-12$

$\Rightarrow \boxed{z=4}$

$\Rightarrow y+z=5 \Rightarrow \boxed{y=1}$

$x+2y+3z=16$

$x+2\,(1)+3\,(4)=16 \Rightarrow x=16-14=2$

$x=2$

$\therefore$ The solution set is (2, 1, 4)

Hence, the correct option is (B).

22. Given $\int_2^3 \frac{1}{1+x^2}\,dx$

$a=2,\ b=3,\ y=\frac{1}{1+x^2},\ n=4$

$\therefore h=\frac{b-a}{n}=\frac{3-2}{4}=\frac{1}{4}=0.25$

x	$y=\frac{1}{1+x^2}$
$x_0=2$	$y_0=\frac{1}{1+2^2}=0.2$
$x_1=2.25$	$y_1=\frac{1}{1+(2.25)^2}=0.1649$
$x_2=2.5$	$y_2=\frac{1}{1+(2.5)^2}=0.1379$
$x_3=2.75$	$y_3=\frac{1}{1+(2.75)^2}=0.1167$
$x_4=3$	$y_3=\frac{1}{1+3^2}=\frac{1}{10}=0.1$

By trapezoidal rule,

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{2}[(y_0+y_n)+2\,(y_1+y_2+\ldots..+y_{n-1})]$$

$$\int_2^3 \frac{1}{1+x^2}\,dx$$

$$=\frac{0.25}{2}[(0.2+0.1)+2\,(0.1649+0.1379+0.1167)]$$

$$=\frac{0.25}{2}[1.139]=0.1423$$

By actual integration, $\int_2^3 \frac{1}{1+x^2}\,dx$

$=(\tan^{-1}x)_2^3=\tan^{-1}3-\tan^{-1}2$

$=1.2490-1.1071=.0.1419$

$\therefore$ Error obtained using Trapezoidal rule

$=$ Exact value $-$ Obtained value

$=0.1419-0.1423=-0.0004$

Hence, the correct option is (D).

23. $a = 0, b = 1, n = 4, h = \frac{b-a}{n} = \frac{1-0}{4} = 0.25$

x	$y = \frac{x^2}{1+8x^3}$
$x_0 = 0$	$y_0 = 0$
$x_1 = 0.25$	$y_1 = \frac{(0.25)^2}{1+8\times(0.25)^3} = 0.0555$
$x_2 = 0.5$	$y_2 = \frac{(0.5)^2}{1+8(0.5)^3} = 0.125$
$x_3 = 0.75$	$y_3 = \frac{(0.75)^2}{1+8(0.75)^3} = 0.1285$
$x_4 = 1$	$y_4 = \frac{1}{1+8} = \frac{1}{9} = 0.1111$

By Trapezoidal rule,

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + \ldots..)]$$

$$\therefore \int_0^1 \frac{x^2}{1+8x^3}dx = \frac{0.25}{2}[(y_0 + y_4) + 2(y_1 + y_2 + y_3)]$$

$$= \frac{1}{8}[(0 + 0.1111) + 2(0.0555 + 0.125 + 0.1285)]$$

$$= \frac{1}{8}[0.1111 + 0.618]$$

$$= \frac{0.7291}{8} = 0.0911$$

Hence, the correct option is (A).

24. In the previous problem, by direct integration

$$\int_0^1 \frac{x^2}{1+8x^3}dx$$

$$= \int_0^1 \frac{x^2}{1+(2x)^3}dx = \frac{1}{24}\int_0^1 \frac{24x^2dx}{1+8x^3}$$

$$= \frac{1}{24}(\log(1 + 8x3))\Big]_0^1 = \frac{1}{24}(\log 9 - \log 1) = 0.0915$$

$\therefore$ Error = Direct value − Trapezoidal value

$= 0.0915 - 0.0911 = 0.0004$

Hence, the correct option is (B).

25. Let $y = x \log x$; $a = 2$, $b = 6$ and $n = 4$

$$\therefore h = \frac{b-a}{n} = \frac{6-2}{4} = 1$$

	x_0	x_1	x_2	x_3	x_4
x	2	3	4	5	6
$x = x\log x$	1.3862	3.2958	5.5451	8.0471	10.7505
	x_0	x_1	x_2	x_3	x_4

$\therefore$ By Simpson's $\frac{1}{3}$ rule,

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{3}[(y_0 + y_n) + 4(y_1 + y_3 + \ldots) + 2(y_2 + y_4 + \ldots.)]$$

$$\therefore \int_2^6 x\log x\,dx = \frac{h}{3}[(y_0 + y_4) + 4(y_1 + y_3) + 2y_2]$$

$$= \frac{1}{3}[(1.3862+10.7505) + 4(3.2958 + 8.0471) + 2(5.5451)]$$

$$= \frac{1}{3}(1.3862 + 10.7505 + 45.3732 + 11.0902)$$

$$= 22.8661$$

Hence, the correct option is (B).

26. Given $\int_0^{\frac{\pi}{2}} \sin x\,dx$

$x_0 = 0, x_n = \frac{\pi}{2}, n = 6, h = \frac{\frac{\pi}{2} - 0}{6} = \frac{\pi}{12}$

X	$y = \sin x$
$x_0 = 0$	$y_0 = \sin 0 = 0$
$x_1 = \frac{\pi}{12}$	$y_1 = \sin\frac{\pi}{12} = 0.258$
$x_2 = \frac{\pi}{6}$	$y_2 = \sin\frac{\pi}{6} = 0.5$
$x_3 = \frac{\pi}{4}$	$y_3 = \sin\frac{\pi}{4} = 0.707$
$x_4 = \frac{\pi}{3}$	$y_3 = \sin\frac{\pi}{3} = 0.8660$
$x_5 = \frac{5\pi}{12}$	$y_5 = \sin\frac{5\pi}{12} = 0.9659$
$x_6 = \frac{\pi}{2}$	$y_6 = \sin\frac{\pi}{2} = 1$

By Simpson's $\frac{1}{3}$ rule,

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{3}[(y_0 + y_n) + 4(y_1 + y_3 + \ldots..) + 2(y_2 + y_4 + \ldots)]$$

$$\therefore \int_0^{\frac{\pi}{2}} \sin x\,dx = \frac{h}{3}[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)]$$

$$= \frac{\pi}{36}[1 + 7.7236 + 2.732] = 0.99968$$

Hence, the correct option is (C).

27. By direct integration, in the previous problem,

$$\int_0^{\frac{\pi}{2}} \sin x dx = (-\cos x)_0^{\frac{\pi}{2}} = -\cos\frac{\pi}{2} - (-\cos 0) = 1$$

$\therefore$ Error = 1 – 0.99968 = 0.00032

Hence, the correct option is (B).

28. Given $\int_0^3 \frac{1}{2+x^2} dx$

Here $a = 0$, $b = 3$, $n = 3$

$$h = \frac{b-a}{n} = \frac{3-0}{3} = 1$$

x	$y = \frac{1}{2+x^2}$
$x_0 = 0$	$y_0 = \frac{1}{2+0^2} = 0.5$
$x_1 = 1$	$y_1 = \frac{1}{2+1} = 0.3333$
$x_2 = 2$	$y_2 = \frac{1}{2+4} = 0.1666$
$x_3 = 3$	$y_3 = \frac{1}{2+9} = 0.909$

By Simpson's $\frac{3}{8}$ rule,

$$\int_{x_0}^{x_n} y dx = \frac{3h}{8}\ [(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 + \ldots.) + 2(y_3 + y_6 + \ldots.)]$$

$$\therefore \int_0^3 \frac{1}{2+x^2} dx = \frac{3h}{8}\ [y_0 + y_3 + 3(y_1 + y_2)]$$

$$= \frac{3}{8}[0.5 + 0.0909 + 3\,(0.3333 + 0.1666)$$

$= 0.7839$

Hence, the correct option is (C).

29. $h = 0.25$,

Volume of the solid $= \int_0^1 \pi y^2 dx$

$$= \frac{h\pi}{3}\ [(y_0^{\ 2} + y_4^{\ 2}) + 4\,(y_1^{\ 2} + y_3^{\ 2}) + 2y_2^{\ 2}]$$

$$= \frac{0.25\pi}{3}[1 + 0.7286 + 4\ (0.9570 + 0.8105) + 2\ (0.8858)]$$

$= 2.7672$

Hence, the correct option is (B).

30. Let $y = \log_e 5$

$\Rightarrow y = \int_0^4 \frac{1}{(1+x)} dx$, here $a = 0$, $b = 4$, $n = 4 \Rightarrow h = 1$

	x_0	x_1	x_2	x_3	x_4
x	0	1	2	3	4
$y = \frac{1}{1+x}$	1 y_0	0.5 y_1	0.33 y_2	0.25 y_3	0.2 y_4

By Simpson's rule,

$$\int_0^4 \frac{1}{1+x} dx = \frac{h}{3}[(y_0 + y_4) + 4\,(y_1 + y_3) + 2y_2]$$

$$= \frac{1}{3}(1 + 0.2 + 4\,(0.5 + 0.25) + 2\,(0.33)]$$

$$= \frac{1}{3}[1.2 + 4\,(0.75) + 0.66] = 1.62$$

Hence, the correct option is (A).

Practice Problems 2

1. Given α, β, γ are the roots of the equation

$x^3 + ax^2 + bx + c = 0$

$\gamma = -1$ From the roots and the coefficients relations

$\alpha + \beta - 1 = -a$ i.e. $-\alpha - \beta + 1 = a$ (13)

$\alpha\beta - \beta - \alpha = b$ (14)

$\alpha\beta\,(-1) = -1$ i.e., $\alpha\beta = c$ (15)

Also given a, b, c are in arithmetic progression

$\Rightarrow 2b = a + c$ (16)

From (13) and (15) $a + c = -\alpha - \beta + 1 + \alpha\beta$

From (16), $2(\alpha\beta - \beta - \alpha) = \alpha\beta - \alpha - \beta + 1$

$\Rightarrow \alpha\beta - \beta - \alpha = 1$

Hence, the correct option is (D).

2. The first equation (A) is $x^3 - 3x^2 + 4 = 0$.

We note that $(-1)^3 - 3(-1)^2 + 4 = 0$.

$\therefore$ By the factor theorem, $x+1$ is a factor of $x^3 - 3x^2 + 4$. Dividing $x^3 - 3x^2 + 4$ by $x+1$, we get $x^2 - 4x + 4$ in the quotient i.e. $x^3 - 3x^2 + 4 = (x + 1)(x - 2)^2$

The second equation (B) is $x^2 + x - 6 = 0$

$\Rightarrow (x + 3)(x - 2) = 0$. The roots of A are –1, 2, 2

The roots of B are –3 and 2.

We see that 2 roots of A are also roots of B, while only are root of B is also a root of A.

Hence, the correct option is (B).

3. The given equation is $x^3 - 7x^2 + 36 = 0$ (17)

Let α, -3α, β be the roots of the equation

$\alpha - 3\alpha + \beta = 7 \Rightarrow \beta - 2\alpha = 7$ (18)

$\alpha(3\alpha) + (-3\alpha)\beta + \beta(\alpha) = 0$

$\Rightarrow -3\alpha^2 - 2\alpha\beta = 0$ (19)

(18), (3) $\Rightarrow -3\alpha^2 - 2\alpha(7 + 2\alpha) = 0$

$\Rightarrow -3\alpha^2 - 14\alpha - 4\alpha^2 = 0$

$\Rightarrow 7\alpha^2 + 14\alpha = 0 \Rightarrow 7\alpha(\alpha + 2) = 0$

$\alpha = 0$, or $\alpha = -2$

$\alpha = -2$ satisfies equation (17) while $\alpha = 0$ does not

$\beta = 7 + 2\alpha = 7 - 4 = 3$

$\therefore$ The roots of the equation are –2, 6, 3

The difference of the greatest two roots = 6 – 3 = 3

Hence, the correct option is (B).

4. Since $f(x) = 0$ has four positive roots, the number of changes of sign in $f(x)$ could be 4, 6, 8, ….. 3 is not the number of sign changes in $f(x) = 0$.

Hence, the correct option is (B).

5. Given $f(x) \equiv ax^3 + bx^3 + cx^2 + dx + e = 0$ has exactly two negative roots. Therefore, there are 2 or 4 sign changes in $f(-x) = ax^4 - bx^3 + cx^2 - dx + e$ we tabulate the four options below

	a	– b	c	– d	e	No.of sign changes
I	+	+	+	–	+	2
II	+	+	+	–	–	1
III	+	+	+	+	+	0
IV	+	+	–	+	–	3

Clearly, options II, III and IV can't be true.

Hence, the correct option is (D).

6. The given equation is

$x^3 + 3x^2 - 10x - 24 = 0$ (20)

Let the roots of the equation be α, 2α, β

$\therefore\ 3\alpha + \beta = -3$ (21)

$\alpha(2\alpha) + 2\alpha\beta + \alpha\beta = -10$

i.e., $2\alpha^2 + 3\alpha\beta = -10$ (22)

and $2\alpha^2\beta = 24$ (23)

(20),(21) $\Rightarrow 2\alpha^2 + 3\alpha(-3 - 3\alpha) + 10 = 0$

$\Rightarrow 2\alpha^2 - 9\alpha - 9\alpha^2 + 10 = 0$

$\Rightarrow 7\alpha^2 + 9\alpha - 10 = 0$

$(\alpha + 2)(7\alpha - 5) = 0$

$\Rightarrow \alpha = -2$ or $\frac{5}{7}$

Let us find β in each case from (21)

When $\alpha = -2$, $\beta = 3$ when $\alpha = 5/7$, $\beta = 36/7$. Only in the first case, (23) is satisfied.

$\therefore\ \alpha = -2$ and $\beta = 3$ i.e., the third root is 3.

Hence, the correct option is (D).

7. By the definition of algebraic equation A is the correct option.

Hence, the correct option is (A).

8. By the definition of transcendental equation B is the correct option.

Hence, the correct option is (B).

9. Let $f(x) = e^2x - 9x$

Since $f(0) = 1 > 0$ and $f(1) = e^2 - 9 = -1.61 < 0$, a root lies between 0 and 1

The first approximation to the root = $\frac{0+1}{2} = 0.5$

Now $f(0.5) = e^{2(0.5)} - 9(0.5)$

$= -1.78 < 0$ and $f(0) > 0$

$\therefore$ The root lies between 0 and 0.5

$\therefore$ The second approximation to the root

$= \frac{0+0.5}{2} = 0.25$

Now, $f(0.25) = e^{2(0.25)} - 9(0.25) = -0.601 < 0$

$\therefore$ The root lies between 0 and 0.25. The third approximation to the root = $\frac{0+0.25}{2} = 0.125$

$f(0.125) = e^{2(0.125)} - 9(0.125)$

$= 0.159 > 0$ and $f(0.25) < 0$

$\therefore$ The root lies between 0.125 and 0.25. The fourth approximation to the root = $\frac{0.125+0.25}{2} = 0.1875$

Hence, the correct option is (B).

10. Since bijection method is the average of this appropriation we can do someway appropriation, $\therefore$ The convergence is very slow

Hence, the correct option is (C).

11. Standard result.

Hence, the correct option is (C).

12. Let $f(x) = 0.32 \sin(0.3 + x) - x$

$f(0) = (0.32)\sin(0.3) = 0.094 > 0$ and $f(1)$

$= (0.32)\sin(1.3) - 1 = -0.691 < 0$

$\therefore$ A root lies between 0 and 1

Using the false position, the first approximation to the root is

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)}, \text{ where } a = 0 \text{ and } b = 1$$

$$= \frac{0 - (1)(0.094)}{-0.691 - 0.094} = \frac{-0.094}{-0.785} = 0.1197$$

Now $f(0.1197) = 0.32 \sin(0.3 + 0.1197) - 0.1197$

$= 0.0106 > 0$ and $f(1) < 0$

$\therefore$ The root lies between 0.1197 and 1

The second approximation to the root

$$x_2 = \frac{0.1197(-0.691)-(1)(0.0106)}{-0.691-0.0106}$$

$$= \frac{-0.0827-0.0106}{-0.7016}$$

$$= \frac{-0.0933}{-0.7016} = 0.1329$$

Now $f(0.1329) = 0.32 \sin(0.3 + 0.1329) - 0.1329$
$= 0.0013416 > 0$ and $f(1) < 0$

∴ The root lies between 0.1329 and 1. The third approximation to the root is

$$x_3 = \frac{(0.1329)(-0.691)-(1)(0.0013)}{-0.691-0.0013}$$

$$= \frac{-0.0931}{-0.6923} = 0.1344$$

Hence, the correct option is (D).

13. Let $f(x) = 5x - 2\cos x - 1$

$f(0) = -3 < 0$ and $f(1) = 5 - 2\cos 1 - 1 = 2.919 > 0$

∴ A root lies between 0 and 1, here a = 0, $b = 1$

$$\therefore x_1 = \frac{af(b)-bf(a)}{f(b)-f(a)}, \frac{0-1(-3)}{2.919-(-3)} = \frac{3}{5.919} = 0.5068$$

$f(0.5068) = 5(0.5068) - 2\cos(0.5068) - 1$
$= -0.2146 < 0$, and $f(1) > 0$

∴ The root lies between 0.5068 and 1

The second approximation x_2,

$$x_2 = \frac{0.5068 f(1) - f(0.5068)}{f(1)-f(0.5068)}$$

$$\Rightarrow x_2 = \frac{(0.5068)(2.919)-(1)(-0.2146)}{2.919-(-0.2146)}$$

$$= \frac{1.6939}{3.1336} = 0.5405$$

Now $f(0.5405) = 5(0.5405) - 2\cos(0.5405) - 1$
$= -0.0124 < 0$

∴ The root lies between 0.5405 and 1

The third approximation to the root,

$$x_3 = \frac{0.5405 f(1)-(1) f(0.5405)}{f(1)-f(0.5405)}$$

$$= \frac{(0.5405)(2.919)-(1)(-0.0124)}{2.919-(-0.0124)}$$

$$= \frac{1.5901}{2.9314} = 0.5424$$

Hence, the correct option is (B).

14. Standard result

Hence, the correct option is (B).

15. Standard result

Hence, the correct option is (B).

16. Standard result

Hence, the correct option is (C).

17. The order of convergence in Newton-Raphson Method = 2

Hence, the correct option is (C).

18. Let $f(x) = x^4 - 2x^3 + x^2 - 3x - 1$

$f(2) = 16 - 16 + 4 - 6 - 1 = -3 < 0$ and

$f(3) = 81 - 54 + 9 - 9 - 1 > 0$

∴ A root lies between 2 and 3

$f^1(x) = 4x^3 - 6x^2 + 2x - 3$

$\Rightarrow f^1(2) = 32 - 24 + 4 - 3$
$= 36 - 27 = 9$

Let $x_0 = 2$

∴ Using Newton's method, the first approximation

$$x_1 = x_0 - \frac{f(x_0)}{f^1(x_0)}$$

$$= 2 - \left(\frac{-3}{9}\right) = 2 + \frac{3}{9} = 2.3333$$

Now $f(2.333)$

$= (2.333)^4 - 2(2.333)^3 + (2.333)^2 - 3(2.333) - 1$

$= 29.6250 - 25.396 + 5.442 - 6.999 - 1$

$= 1.672 > 0$

$f^1(x_1) = 4(2.333)^3 - 6(2.333)^2 + 2(2.333) - 3 = 19.801$

The second approximation

$$x_2 = x_1 - \frac{f(x_1)}{f^1(x_1)} = (2.333) - \frac{1.672}{19.801} = 2.248$$

Hence, the correct option is (A).

19. $N = 52$, $x_0 = 7.5$; to find $\sqrt{N}$, using Newton's iterative formula, $\sqrt[b]{a}$'s iteration formula is

$$x_{n+1} = \frac{1}{b}\left\{(b-1)x_n + \frac{a}{x_n^{\,b-1}}\right\}$$

Put $a = 52$ and $b = 2$

$$x_1 = \frac{1}{2}\left\{x_0 + \frac{52}{x_0}\right\} = \frac{1}{2}\left\{7.5 + \frac{52}{7.5}\right\} = 7.216$$

Hence, the correct option is (B).

20. $P = 30$, $x_0 = 3.5$, to find $\sqrt[3]{N}$, using Newton's iteration formula, to find $\sqrt[b]{a}$

$$x_{n+1} = \frac{1}{b}\left\{(b-1)x_n + \frac{a}{x_n^{\,b-1}}\right\}$$

Put $a = 30$, $b = 3$

$$x_1 = \frac{1}{3}\left\{2x_0 + \frac{30}{x_0^{\,2}}\right\}$$

$$= \frac{1}{3}\left\{(2\times 3.5)+\frac{30}{3.5^2}\right\} = \frac{1}{3}\left\{7+\frac{30}{12.25}\right\} = 3.1496$$

$$x_2 = \frac{1}{3}\left\{2x_1+\frac{30}{x_1^2}\right\} = \frac{1}{3}\left\{2(3.1496)+\frac{30}{(3.1496)^2}\right\}$$

$$= \frac{1}{3}\left\{6.2992+\frac{30}{9.9199}\right\} = 3.1078$$

Hence, the correct option is (B).

21. Given: $\left(\frac{1}{5}\right)^{1/6}$

We know that, using Newton's iteration formula, to find $\sqrt[b]{a}$,

$$x_{n+1} = \frac{1}{b}\left\{(b-1)x_n+\frac{a}{x_n^{b-1}}\right\}$$

Put $b = 6$ and a $= \frac{1}{5}$ and let $x_0 = 0.5$ we get

$$x_1 = \frac{1}{6}\left\{5x_0+\frac{\frac{1}{5}}{x_0^5}\right\}$$

$$= \frac{1}{6}\left\{5(0.5)+\frac{1}{5(0.5)^5}\right\} = \frac{1}{6}\{2.5+6.4\} = 1.483$$

$$x_2 = \frac{1}{6}\left\{5x_1+\frac{1}{5x_1^5}\right\} = \frac{1}{6}\left\{5(1.483)+\frac{1}{5(1.483)^5}\right\} = 1.2404$$

$$x_3 = \frac{1}{6}\left(5(1.2404)+\frac{1}{5(1.2404)^5}\right) = 1.0450$$

$$x_4 = \frac{1}{6}\left\{5(1.0450)+\frac{1}{5(1.0450)^5}\right\} = 0.8975$$

$$x_5 = \frac{1}{6}\left\{5(0.8975)+\frac{1}{5(0.8975)^5}\right\} = 0.8051$$

Hence, the correct option is (A).

22. Let $f(x) = 2x - \cos x$

$f(0.5) = (0.5\times 2) - \cos(0.5) = 1 - 0.8775 = 0.1224 > 0$

and $f(0) = 0 - \cos 0 = -1 < 0$

$\therefore$ The root lies between 0 and 0.5

The first approximation to the required root

$$= \frac{0+0.5}{2}$$

$= 0.25$

Now $f(0.25) = 2(0.25) - \cos(0.25)$

$= 0.5 - 0.9689 = -0.4689 < 0$

$\therefore$ The root lies between 0.25 and 0.5

$\therefore$ The second approximation to the required root

$$= \frac{0.25+0.5}{2} = \frac{0.75}{2} = 0.375$$

Hence, the correct option is (C).

23. Standard result

Hence, the correct option is (D).

24. Standard result

Hence, the correct option is (B).

25. Standard result

Hence, the correct option is (D).

26. Standard result

Hence, the correct option is (C).

27. Given $\int_5^7 \frac{5}{3+x^2}dx$, here $a = 5, b = 7, n = 8$

$\therefore h = \frac{b-a}{n} = \frac{7-5}{8}$, $x_0 = 5$ and $y_0 = 0.1785$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
x	5.25	5.5	5.75	6	6.25	6.5	6.75	7
$y = \frac{5}{3+x^2}$	0.1635 y_1	0.1503 y_2	0.1386 y_3	0.1282 y_4	0.1188 y_5	0.1104 y_6	0.1029 y_7	0.0961 y_8

By trapezoidal rule, $\int_5^7 \frac{5}{3+x^2}dx = \frac{h}{2}[(y_0+y_8)+2(y_1+y_2+y_3+y_4+y_5+y_6+y_7)]$

$$= \frac{0.25}{2}[(0.1785+0.0961)+2(0.1635)+0.1503+0.1386+0.1282+0.1188+0.1104+0.1029) = 0.2625$$

Hence, the correct option is (D).

28. Standard result

Hence, the correct option is (C).

29. Standard result

Hence, the correct option is (A).

30. Standard result

Hence, the correct option is (A).

31. Given: $\int_0^6 2e^x dx$

$a = 0, b = 6, n = 6 \Rightarrow h = \frac{b-a}{n} = \frac{6-0}{6} = 1$

	x_0	x_1	x_2	x_3	x_4	x_5	x_6
x	0	1	2	3	4	5	6
$y = 2e^x$	2 y_0	5.436 y_1	14.778 y_2	40.171 y_3	109.196 y_4	296.826 y_5	806.857 y_6

By Simpson's $\frac{1}{3}$ rd rule,

$$\int_0^6 2e^x dx = \frac{h}{3}[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)]$$

$$= \frac{1}{3}[(2 + 806.857) + 4(5.436 + 40.171 + 296.826) + 2(14.778 + 109.196)]$$

$$= \frac{1}{3}[2 + 806.857 + 1369.732 + 2(14.778 + 109.196)]$$

$$= 808.845$$

By actual integration,

$$\int_0^6 2e^x dx = 2e^x]_0^6 \Rightarrow 2[e^6 - e^0] = 2[402.428] = 804.856$$

$\therefore$ Error obtained = 804.856 – 808.845

= –3.989

Hence, the correct option is (C).

32. Standard result

Hence, the correct option is (C).

33. Standard result

Hence, the correct option is (B).

34. $y = f(x) = \sqrt{1+x^3}$,

$h = 0.5$

	x_0	x_1	x_2	x_3	x_4	x_5	x_6
x	1	1.5	2	2.5	3	3.5	4
$Y = \sqrt{1+x^3}$	1.414 y_0	2.091 y_1	3 y_2	4.077 y_3	5.291 y_4	6.623 y_5	8.062 y_6

By Simpson's $\frac{3}{8}$ th rule,

$$\int_1^4 \sqrt{1+x^3} dx = \frac{3h}{8}[(y_0 + y_6) + 3(y_1 + y_2 + y_4 + y_5) + 2y_3]$$

$$= \frac{3(0.5)}{8}[(1.414 + 8.062) + 3(2.091 + 3 + 5.291 + 6.623) + 2(4.077)]$$

$$= \frac{1.5}{8}[9.476 + 3(17.005) + 8.154]$$

$$= 12.8709$$

Hence, the correct option is (B).

35. Standard result

Hence, the correct option is (B).

Previous Years' Questions

1. Let $f(x) = 0.75x^3 - 2x^2 - 2x + 4 = 0$

Given $x_0 = 2$

$f^1(x) = 2.25x^2 - 4x - 2$

$f(x_0) = f(2) = -2$ and $f^1(x_0) = f^1(2) = -1$

By Newton – Raphson method,

$$x_1 = x_0 - \frac{f(x_0)}{f^1(x_0)} = 2 - \frac{(-2)}{(-1)} = 0$$

Now $f(x_1) = f(0) = 4$ and $f^1(x_1) = f^1(0) = -2$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f^1(x_1)} = 0 - \frac{4}{(-2)} = 2 \qquad \therefore x_2 = 2$$

$f(x_2) = f(2) = -2$ and $f^1(x_2) = f^1(2) = -1$

$$\therefore x_3 = x_2 - \frac{f(x_2)}{f^1(x_2)} = 2 - \frac{(-2)}{(-1)} = 0$$

$\therefore x_3 = 0$ (24)

From (24), statement (I) is TRUE (25)

Also, one can easily observe that by Newton-Raphson method, we get

$x_4 = 2, x_5 = 0, x_6 = 2, x_7 = 0,$

Hence with $x_0 = 2$, the method is NOT converging to a solution.

$\therefore$ Statement (II) is NOT TRUE (26)

Hence from (25) and (26) only the statement (I) is TRUE.

Hence, the correct option is (A).

2. Given $k = \int_a^b x^2 dx$.

We know that the value of the definite integral obtained by Simpson's rule coincides with the actual value of the integral upto the second degree polynomial.

Hence the statement (II) is TRUE.

One can easily observe that the value of the definite integral obtained by Trapezoidal Rule for a second degree polynomial exceeds the actual value of that integral. So the statement (I) is also TRUE.

Hence, the correct option is (C).

3. By trapezoidal rule

$$= \frac{h}{2}\left[y_0 + y_n + 2\left(y_1 + y_2 + \ldots\ldots + y_{n-1}\right)\right]$$

Given $h = 0.3$, $n = 10$

$$\int_0^3 f(x)\,dx = \frac{0.3}{2}\ [0 + 9 + 2(0.09 + 0.36 + 0.81 + 1.44 + 2.25 + 3.24 + 4.41 + 5.76 + 7.29)]$$

$$= \frac{0.3}{2}\ [9 + 51.3] = \frac{0.3}{2} \times 60.3 = 9.045$$

Hence, the correct option is (D).

4. $f(x) = x^4 - x^3 - x^2 - 4$

$f(1) = 1 - 1 - 1 - 4 = -5 < 0$

$f(9) = 9^4 - 9^3 - 9^2 - 4 = +ve > 0$

One root lies between [1, 9].

First iteration in $[a, b]$ is $\frac{a+b}{2}$.

$$\frac{1+9}{2} = 5$$

$f(5) = 5^4 - 5^3 - 5^2 - 4 = +ve > 0$

$\therefore$ One root lies between [1,5].

Second iteration in [1,5] is $\frac{1+5}{2} = 3$

$f(3) = 3^4 - 3^3 - 3^2 - 4 = +ve$

One root is lies between [1, 3].

Third iteration in [1, 3] $= \frac{1+3}{2} = 2$

$f(2) = 2^4 - 2^3 - 2^2 - 4 = 0$

$\therefore$ After three iterations $f(x) = 0$

Hence, the correct option is (B).

5. Let $f(x) = x^2 - 13 \Rightarrow f^1(x) = 2x$

Initial value of $x = x_0 = 3.5$

By Newton-Raphson method,

$$x_1 = x_0 - \frac{f(x_0)}{f^1(x_0)} = x_0 - \frac{\left(x_0^{\,2} - 13\right)}{2x_0}$$

$$= 3.5 - \frac{\left|(3.5)^2 - 13\right|}{2\times 3.5}$$

$\therefore x_1 = 3.607$

Hence, the correct option is (D).

6. In trapezoidal rule, maximum error $= \left|\frac{(b-a)h^2}{12}M\right|$

Where $M = \max\{y_1^{11}, y_2^{12}, \ldots\ldots\}$

Here $y = f(x) = xe^x$

$a = 1$ and $b = 2$

$\Rightarrow y^1 = (x + 1)ex$ and $y^{11} = (x + 2)e^x$

And y^{11} will have the max. value at $x = 2$

$\therefore M = y^{11}$ at $x = 2 = 4e^2$

Now we have to find the value of $\int_1^2 xe^x\,dx$ to an accuracy of atleast $\frac{1}{3}\times 10^{-6}$ means,

The maximum error in $\int_1^2 x\,e^x\,dx \le \frac{1}{3}\times 10^{-6}$

$$\Rightarrow \frac{(2-1)h^2}{12}4e^2 \le \frac{1}{3}\times 10^{-6} \Rightarrow h^2 e^2 \le 10^{-6}$$

$$\Rightarrow h^2 \le \frac{10^{-6}}{e^2} \Rightarrow h \le \frac{10^{-3}}{e}$$

$$\Rightarrow \frac{b-a}{n} = \frac{1}{1000e}$$

$$\Rightarrow \frac{1}{n} \le \frac{1}{1000e} \Rightarrow n \ge 1000e$$

$\therefore$ The minimum no of equal intervals $= n = 1000e$

Hence, the correct option is (A).

7. The Newton-Raphson iteration formula to find K^{th} root of a positive real number R is

$$X_{n+1} = \frac{(K-1)x_n^{\,K} + R}{Kx_n^{\,K-1}}$$

Take $K = 2$,

$$X_{n+1} = \frac{x_n^{\,2} + R}{2x_n}$$

$$\therefore x_{n+1} = \frac{1}{2}\left(x_n + \frac{R}{x_n}\right)$$

So, the given iteration formula can be used to compute the square root of R.

Hence, the correct option is (C).

8. Given $x_{n+1} = \frac{x_n}{2} + \frac{9}{8x_n}$ and $x_0 = 0.5$

$$x_1 = \frac{x_0}{2} + \frac{9}{8x_0} = 2.5$$

$$x_2 = \frac{x_1}{2} + \frac{9}{8x_1} = 1.7$$

$$x_3 = \frac{x_2}{2} + \frac{9}{8x_2} = 1.5009$$

$$x_4 = \frac{x_3}{2} + \frac{9}{8x_3} = 1.5004$$

$$x_5 = \frac{x_4}{2} + \frac{9}{8x_4} = 1.5000$$

Hence the series converges to 1.5

Hence, the correct option is (A).